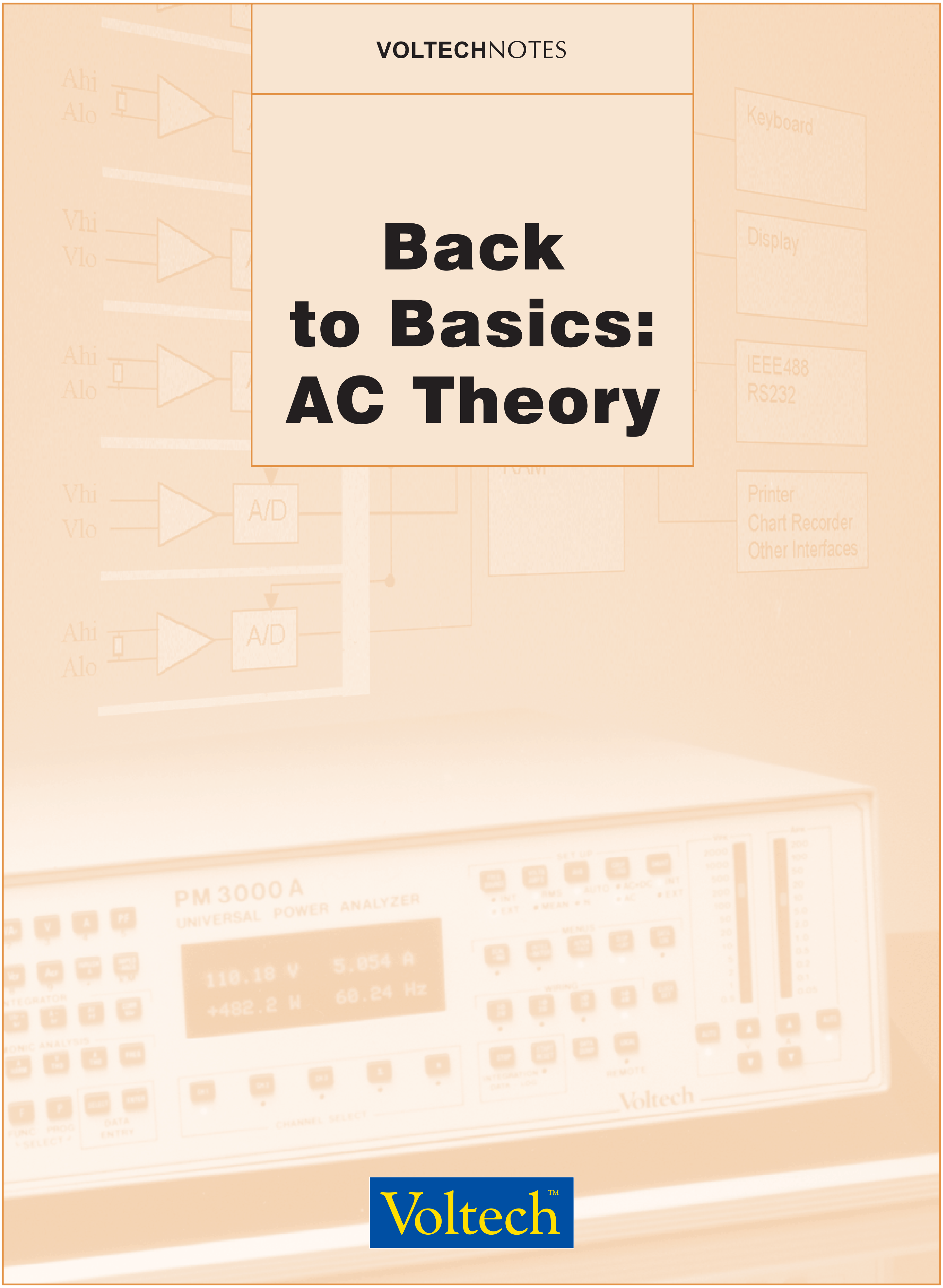
VOLTECHNOTES

Back to Basics: AC Theory



Voltech™

VPN 104-109/3

BACK TO BASICS: AC THEORY

The complex current and voltage waveforms associated with many of today's AC power applications pose specific measurement problems. A good starting point for tackling some of these problems is an understanding of the basic measurements, the terms used and the relationships between them.

RMS (Root mean squared value)

The RMS value is the most commonly used and useful means of specifying the value of both AC voltage and current. The RMS value of an AC waveform indicates the level of power that is available from that waveform, this being one of the most important attributes of any AC source.

The calculation of an RMS value can best be described by considering an AC current waveform and its associated heating effect such as that shown in Figure 1a below.

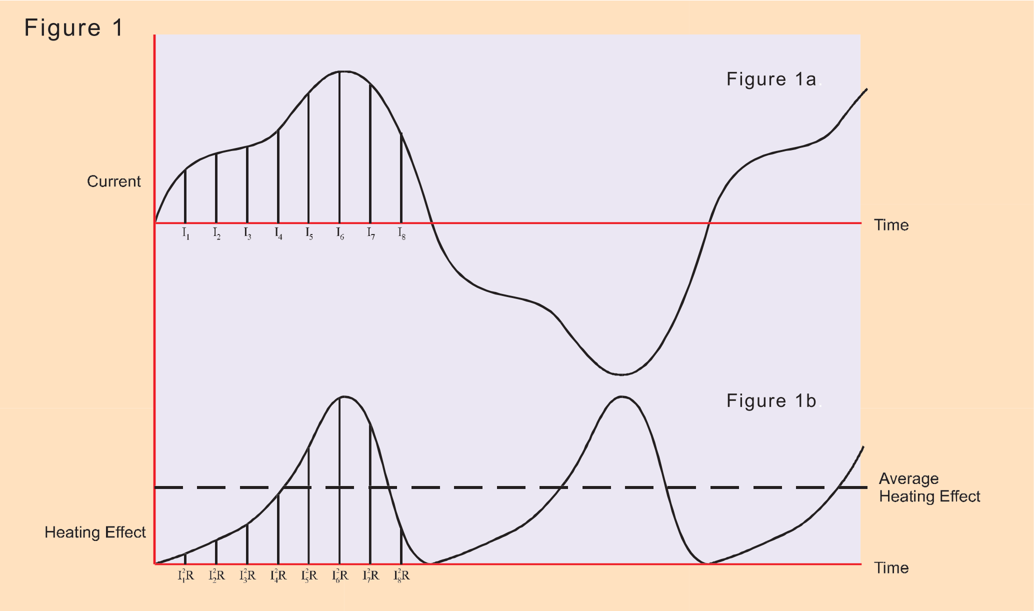


Figure 1

If this current is considered to be flowing through a resistance, the heating effect at any instant is given by the equation:

$$W = I^2R$$

By dividing the current cycle into equally spaced coordinates, the variation of the heating effect with time can be determined as shown in Figure 1b above.

The average heating effect (power) is given by:

$$W = \frac{I_1^2R + I_2^2R + I_3^2R..... + I_n^2R}{n}$$

If we wanted to find the equivalent value of current that would produce the average heating effect value shown above, then the following applies:

$$I^2R = \frac{I_1^2R + I_2^2R + I_3^2R..... + I_n^2R}{n}$$

Therefore:

$$I = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2..... + I_n^2}{n}}$$

= the square root of the mean of the squares of the current

= the RMS value of the current.

This value is often termed the effective value of the AC waveform, as it is equivalent to the direct current that produces the same heating effect (power) in the resistive load.

It is worth noting that for a sinusoidal waveform:

$$RMS\ value = \frac{peak\ value}{\sqrt{2}} \qquad ie:\ RMS = 0.707\ x\ peak\ value$$

Average value

The average value of a waveform such as that shown in Figure 2 is given by:

$$Average\ Value = \frac{Area\ enclosed\ by\ one\ half\ cycle}{Length\ of\ base\ over\ one\ half\ cycle}$$

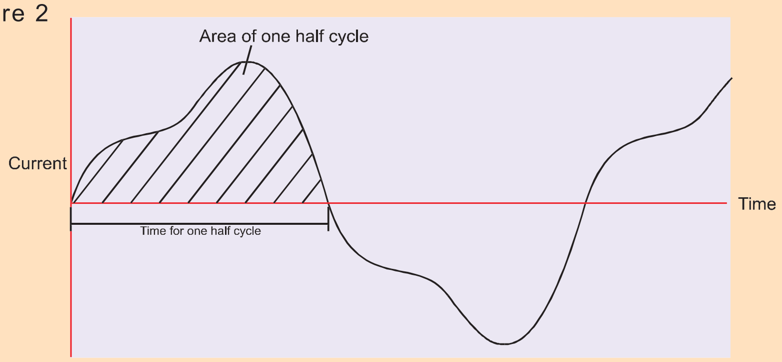
It is clear the average value can only have real meaning over one half cycle of the waveform since, for a symmetrical waveform, the mean or average value over a complete cycle is zero. Most simple multimeters determine AC values by full-wave rectification of the AC waveform, followed by a calculation of the mean value.

Such meters, however, will be calibrated in RMS and will make use of the known relationship between RMS and average for a sinusoidal waveform, i.e.

$$RMS = 1.11 \times mean$$

However, for waveforms other than a pure sinewave, the readings from such meters will be invalid.

Figure 2



Real and apparent power (W & VA)

If a sinusoidal voltage source of, say, 100V RMS is connected to a resistive load of, say, 100ohms, then the voltage and current can be depicted as in Figure 3a and are said to be “in phase”.

The power that flows from the supply to the load at any instant is given by the value of the product of the voltage and the current at that instant, as illustrated in Figure 3b.

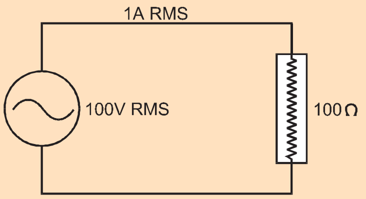
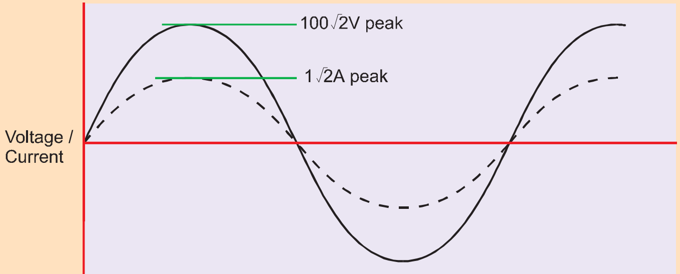


Figure 3a



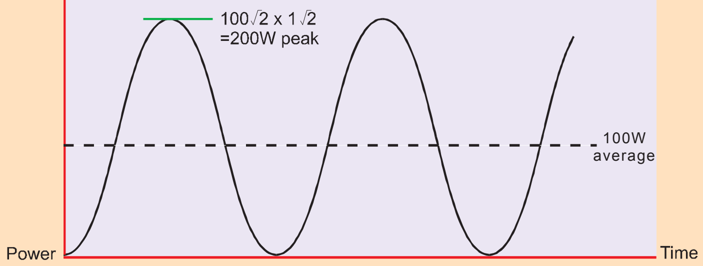


Figure 3b

From this, it can be seen that the power flowing into the load fluctuates (at twice the supply frequency) between 0 and 200W and that the average power delivered to the load equals 100W—which is what one might expect from 100V RMS and a resistance of 100ohms.

However, if the load is reactive (i.e. contains inductance or capacitance as well as resistance) with an impedance of 100ohms, then the current that flows will still be 1A RMS but will no longer be in-phase with the voltage. This is shown in Figure 4a for an inductive load with the current lagging by 60°.

Although the power flow continues to fluctuate at twice the supply frequency, it now flows from the supply to the load during only a part of each half cycle—during the remaining part, it actually flows from the load to the supply.

The average net flow into the load, therefore, is much smaller than in the case of a resistive load as shown in Figure 4b—with only 50W of useful powering delivered into the inductive load.

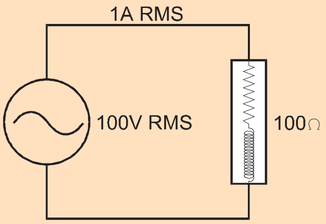


Figure 4a

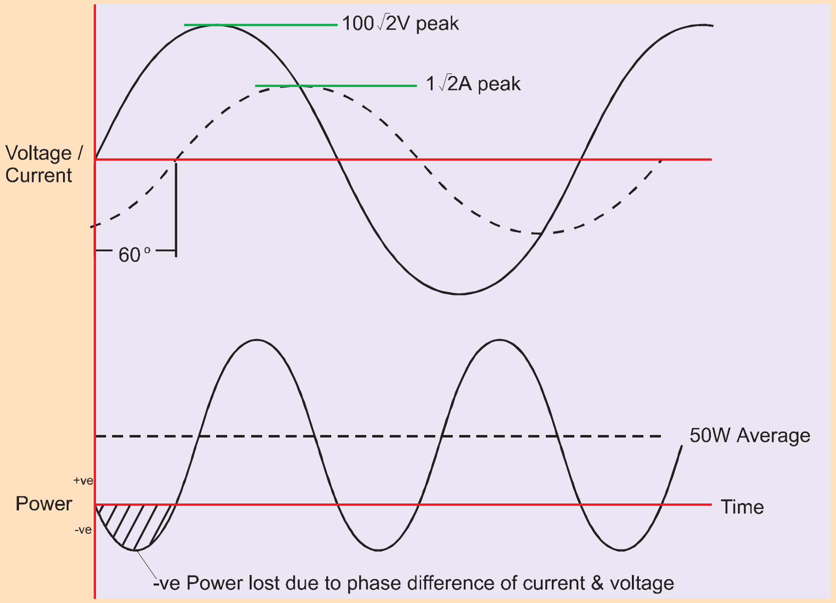


Figure 4b

In both of the above cases the RMS voltage was equal to 100V RMS and the current was 1A RMS . The product of these two values is the apparent power delivered into the load and is measured in VA as follows:

$$Apparent\ Power = V_{RMS} \times A_{RMS}$$

The real power delivered has been shown to depend on the nature of the load. It is not possible to determine the value of real power from the knowledge of RMS voltage and current. This can only be achieved (e.g., for assessing heat loss or efficiency) through the use of a true AC power meter, capable of computing the product of the instantaneous voltage and current values and displaying the mean of the result.

Power factor

It is clear that, in comparison with DC systems, the transferred AC power is not simply the product of the voltage and current values. A further element known as the power factor must also be taken into consideration. In the previous example (real and apparent power) with an inductive load, the power factor is 0.5 because the useful power is exactly one half of the apparent power. We can therefore define power factor as:

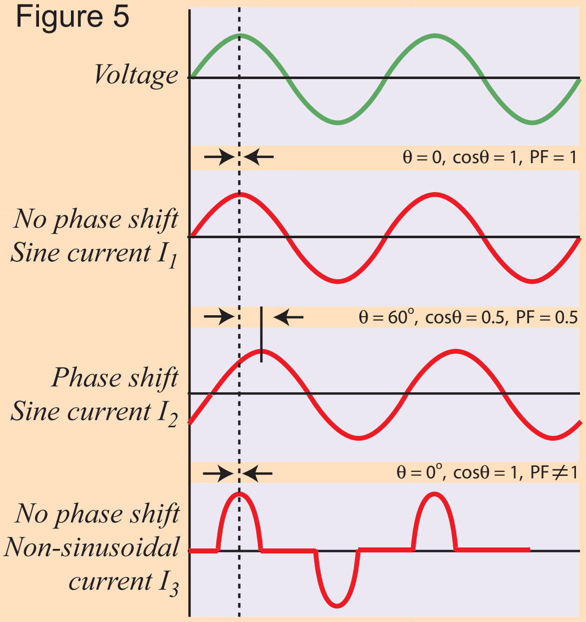
$$Power\ factor = \frac{Real\ power}{Apparent\ power}$$

In the case of sinusoidal voltage and current waveforms, the power factor is actually equal to the cosine of the phase angle (θ) between the voltage and current waveforms. For example, with the inductive load described earlier, the current lags the voltage by 60^{o}.
Therefore:

$$PF = cos\theta = cos60 = 0.5$$

It is for this reason that power factor is often referred to as cosθ. It is, however, important to remember that this is only the case when both voltage and current are sinusoidal [Figure 5 (I_1 and I_2)] and that power factor is not equal to cosθ in any other case [Figure 5 (I_3)]. This must be remembered when using a power factor meter that reads cosθ, as the reading will not be valid except for pure sinusoidal voltage and current waveforms. A true power factor meter will compute the ratio of real to apparent power as described above.

Figure 5



Crest factor

It has already been shown that for a sinusoidal waveform:

$$\text{Peak value} = \text{RMS } x \sqrt{2}$$

The relationship between peak and RMS is known as the crest factor and is defined as:

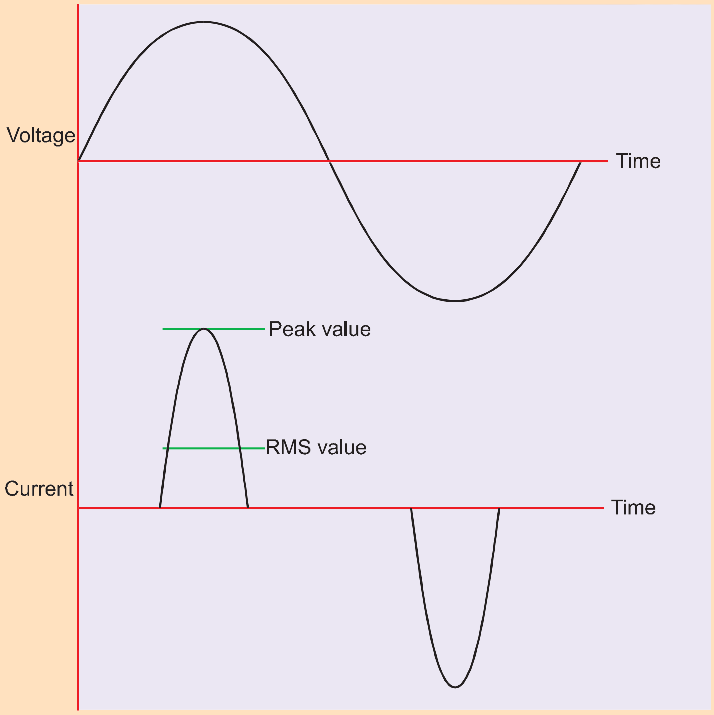
$$\text{Crest factor} = \frac{\text{Peak value}}{\text{RMS value}}$$

Thus, for a sinusoid:

$$\text{Crest factor} = \sqrt{2} \cong 1.41$$

Many items of modern equipment connected to the AC supply take non-sinusoidal current waveforms. These include power supplies, lamp dimmers, and even fluorescent lamps.

Figure 6



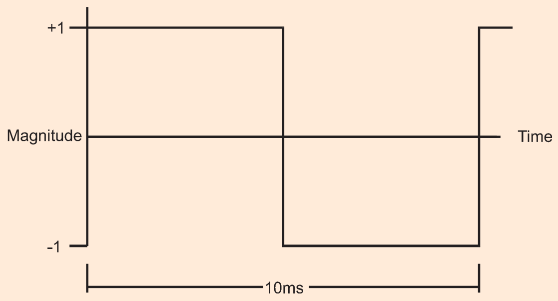
A typical switch-mode power supply (SMPS) will take current from the AC supply as shown in Figure 6. It is clear that the crest factor of the current waveform depicted is much greater than 1.414—indeed, most switch-mode power supplies and motor speed controllers have a current crest factor of 3 or greater. It follows therefore, that a large current crest factor must put additional stress on equipment supplying such a load, as the equipment must be capable of supplying the large peak currents associated with the distorted waveform. This is particularly relevant where a limited impedance power source, such as a standby inverter, is supplying the load. It is thus clear that, where AC equipment is involved, it is important to know the crest factor of the current drawn as well as its RMS current.

Harmonic distortion

If a load introduces distortion of the current waveform, it is useful, in addition to knowing the crest factor, to quantify the level of distortion of the waveshape. Observation on an oscilloscope will indicate distortion but not the level of distortion.

It can be shown by Fourier analysis that a non-sinusoidal current waveform consists of a fundamental component at the supply frequency plus a series of harmonics (i.e. components at frequencies that are integral multiples of the supply frequency). For example, a 100Hz square wave consists of the components shown in Figure 7. A square wave is clearly very distorted compared to a pure sine wave. However, the current waveform drawn by, for example, a SMPS, a lamp dimmer or even a speed-controlled washing machine motor can contain harmonics of even greater significance. Figure 8 shows the current drawn by a popular SMPS model together with the harmonic content of that current.

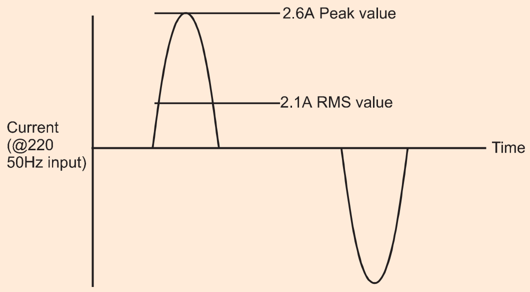
Figure 7



FREQUENCY	MAGNITUDE
100Hz (Fundamental)	1 x 4/π
300Hz (3rd Harmonic)	⅓ x 4/π
500Hz (5th Harmonic)	⅕ x 4/π
700Hz (7th Harmonic)	⅐ x 4/π
▼ ▼ ▼	▼ ▼ ▼

Note: There are no even harmonics as both half cycles are symmetrical.

Figure 8



FREQUENCY	MAGNITUDE
50Hz (Fundamental)	1.40A rms
150Hz (3rd Harmonic)	1.21A rms
250Hz (5th Harmonic)	0.85A rms
350Hz (7th Harmonic)	0.48A rms
▼ ▼ ▼	▼ ▼ ▼

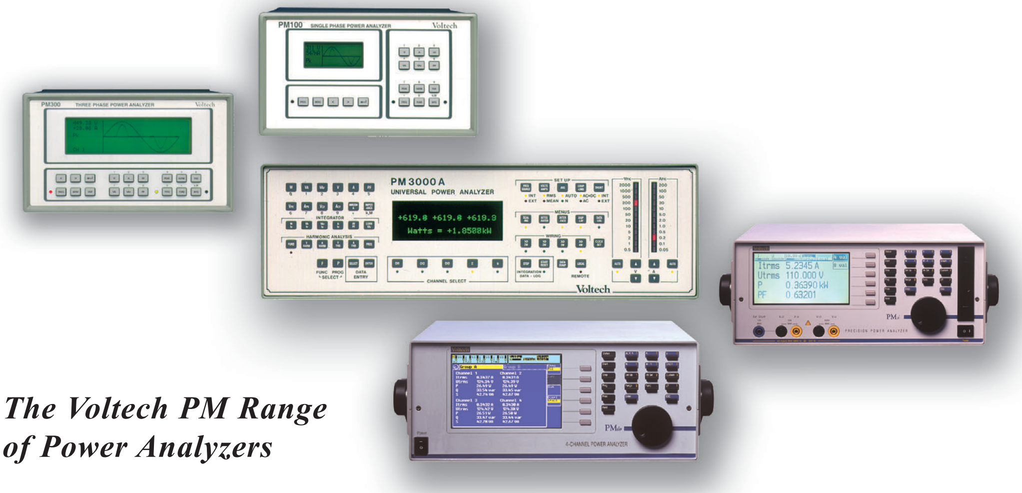
The only useful current is the fundamental component of current, as it is only this that can generate useful power. The additional harmonic current not only flows within the power supply itself, but in all of the distribution cables, transformers and switchgear associated with the power supply and will thus cause additional loss.

There is an increasing awareness of the need to limit the level of harmonics that equipment can produce. Controls exist in many territories to provide mandatory limits on the level of harmonic current permitted for certain types of load. Such regulatory controls are becoming more widespread with the use of internationally recognized standards such as EN61000-3. Thus, there is a need for an increased awareness amongst equipment designers as to whether their products generate harmonics and at what level.

Measurement of AC parameters

It has been shown that the AC parameters described above can be important to both equipment manufacturers and the suppliers of AC power. It is common, however, that the instrumentation being utilized for these applications is either inconvenient to use or incapable of providing the required functionality or accuracy, especially when the signals being analyzed are noisy or distorted.

Voltech Instruments specialize in the development and manufacture of power measurement instrumentation designed to provide solutions for a wide range of applications from general purpose power measurement to the most complex and demanding power analysis tasks.



The Voltech PM Range of Power Analyzers

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